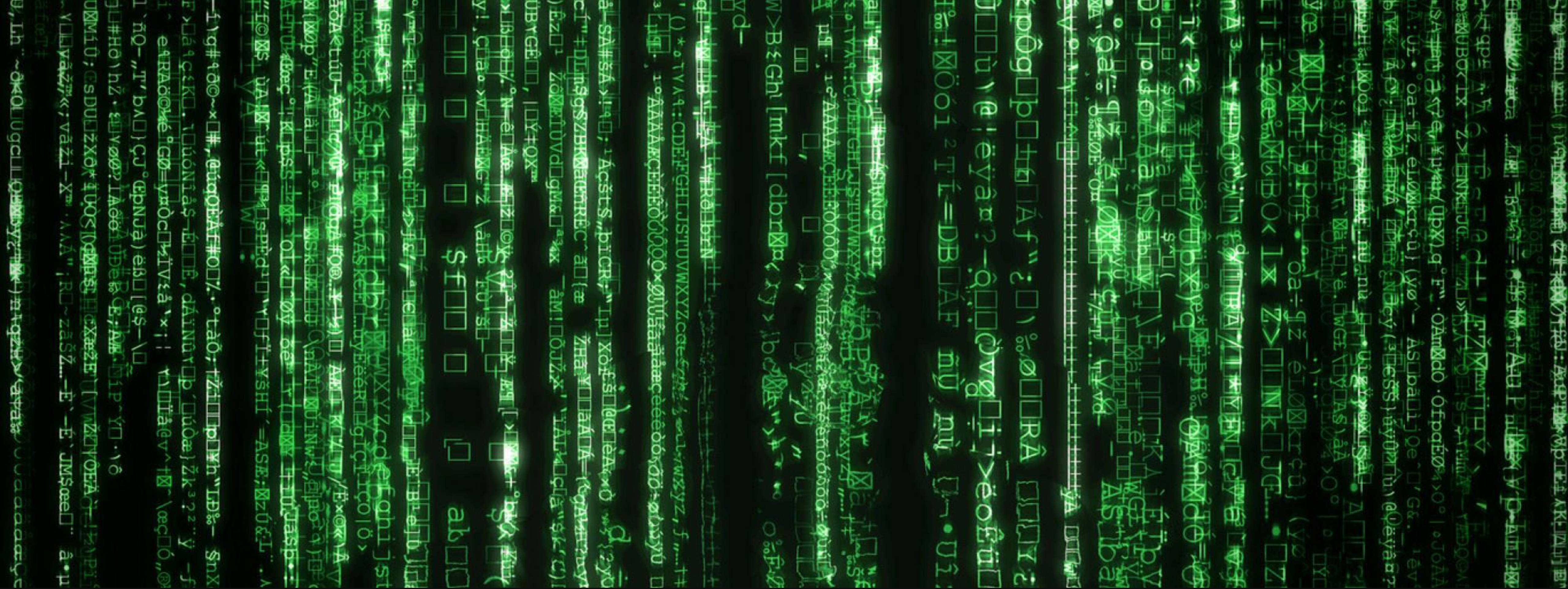

Determinants & Cramer’s Rule

Section 4.3

Evaluating Determinants

Associated with each <u>square</u> matrix is a real number called its **determinant**. The determinant of matrix A is denoted by

$\det A$ or by $|A|$

Determinant of a 2 x 2 Matrix

The determinant of a 2 x 2 matrix is the difference of the products of the entries of the diagonals

$$\underbrace{\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}}_{\text{Notation}} = ad - cb$$

Example: Find the determinant of the matrix $\begin{bmatrix} 3 & -1 \\ -6 & 7 \end{bmatrix}$

$(3)(7) - (-6)(-1)$

$21 - 6 = \boxed{15}$

Determinant of a 3 x 3 Matrix

Steps to follow

1. Repeat the first 2 columns to the right of the determinant
2. Subtract the sum of the products in red (bottom to top) from the sum of the products in blue (top to bottom)

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right|\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (\boldsymbol{aei} + \boldsymbol{bfg} + \boldsymbol{cdh}) - (\boldsymbol{gec} + \boldsymbol{hfa} + \boldsymbol{idb})$$

Determinant of a 3 x 3 Matrix

Directions: Find the determinant of the matrices:

$$\begin{bmatrix} -3 & -1 & 1 \\ 2 & 5 & 6 \\ 2 & 4 & -2 \end{bmatrix} \begin{matrix} -3 & -1 \\ 2 & 5 \\ 2 & 4 \end{matrix}$$

$$\Big((-3)(5)(-2) + (-1)(6)(2) + (1)(2)(4)\Big) - \Big((2)(5)(1) + (4)(6)(-3) + (-2)(2)(-1)\Big)$$

$$(30 + -12 + 8) \quad - \quad 10 + -72 + 4$$

$$26 \quad - \quad -58$$

$|A| = 84$

$\det A = 84$

$$\begin{bmatrix} 11 & 5 & 3 \\ 7 & -4 & -1 \\ -8 & 0 & 2 \end{bmatrix} \begin{matrix} 11 & 5 \\ 7 & -4 \\ -8 & 0 \end{matrix}$$

$$(-88 + 40 + 0) - (96 + 0 + 70)$$

$$-48 - 166 = \boxed{-214}$$

Cramer’s Rule

You can use determinants to solve a system of linear equations. The method, called Cramer’s rule, and named after the Swiss mathematician Gabriel Cramer (1704 – 1752), uses the coefficient matrix of the linear system.

Using Cramer's Rule for a 2 x 2 System

- Let A be the coefficient matrix of this linear system: * standard form

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

- If $\det A \neq 0$, then the system has exactly one solution. The solution is:

constants   y-coeff.   x coeff.   constants

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Use Cramer's Rule to Solve the System

$3x - 2y = 22$
$x + 4y = -2$

Coefficient Matrix (A)

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \qquad |A| = 12 - -2 = 14$$

det

$$x = \frac{\begin{vmatrix} 22 & -2 \\ -2 & 4 \end{vmatrix}}{14} = \frac{88-4}{14} = \frac{84}{14} = 6$$

$$y = \frac{\begin{vmatrix} 3 & 22 \\ 1 & -2 \end{vmatrix}}{14} = \frac{-6-22}{14} = \frac{-28}{14} = -2$$

FINAL ANSWER
$(6,-2)$

$5x - 2y = -9$
$-7x + 3y = 14$

$$|A| = \begin{vmatrix} 5 & -2 \\ -7 & 3 \end{vmatrix} = 15 - 14 = \boxed{1}$$

$$x = \frac{\begin{vmatrix} -9 & -2 \\ 14 & 3 \end{vmatrix}}{1} = -27 + 28 = 1$$

$$y = \frac{\begin{vmatrix} 5 & -9 \\ -7 & 14 \end{vmatrix}}{1} = 70 - 63 = 7$$

$$\boxed{(1,7)}$$

Using Cramer's Rule for a 3 x 3 System

- Let A be the coefficient matrix of this linear system:

$$\begin{gathered} ax + by + cz = j \\ dx + ey + fz = k \\ gx + hy + iz = l \end{gathered}$$

- If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A} \quad , \quad \mathrm{y} = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A} \quad , \textit{and} \quad \mathrm{z} = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$$

Use Cramer's Rule to Solve the System

$2x - 2y + 5z = 1$
$-8x + z = -6$
$x + y - 2z = 1$

$$|A| = \left|\begin{matrix} 2 & -2 & 5 \\ -8 & 0 & 1 \\ 1 & 1 & -2 \end{matrix}\right| \begin{matrix} 2 & -2 \\ -8 & 0 \\ 1 & 1 \end{matrix}$$

$(0 + -2 + -40) - (0 + 2 + -32)$

$-42 - -30 = \boxed{-12} = |A|$

Use Cramer's Rule to Solve the System

$$2x + y - 3z = 0$$
$$3x - 2y + z = -7$$
$$2x + 2y - z = 2$$

Nutrition

For lunch you eat a peanut butter sandwich on wheat bread and carrot sticks. The nutritional content of peanut butter, wheat bread, and carrots is shown in the table below. Use Cramer's rule to determine how many calories are in a gram of carbohydrates, fat, and protein.

Serving	Carbohydrates per serving	Fat per serving	Protein per serving	Calories per serving
Peanut Butter	7 g	16 g	8 g	204
Wheat Bread (2 slices)	26 g	1 g	6 g	137
Carrots	8 g	0 g	1 g	36

Practice

- Textbook page 218 #12, 14, 22, 24
- Textbook page 219 #36, 38, 46